How fast could Usain Bolt have run? A dynamical study

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Since that memorable day at the Beijing 2008 Olympics, a big question has been, "What would the 100 m dash world record have been had Usain Bolt not celebrated at the end of his race?" Bolt's coach suggested that the time could have been 9.52 s or better. We consider this question by measuring Bolt's position as a function of time using footage of the run, and then extrapolate the last 2 s with two different assumptions. First, we conservatively assume that Bolt could have maintained the runner-up's acceleration during the end of the race. Second, based on the race development prior to the celebration, we assume that Bolt could have kept an acceleration of 0.5 $\mathrm{m/s^2}$ greater than the runner-up. We find that the new world record in these two cases would have been 9.61 ± 0.04 and 9.55 ± 0.04 s, respectively, where the uncertainties denote 95% statistical errors.

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I. INTRODUCTION

On Saturday, 16 August 2008, Usain Bolt shattered the world record of the 100 m dash at the Beijing Olympics 2008. In a spectacular run dubbed, "the greatest 100 meter performance in the history of the event" by Michael Johnson, Bolt finished at 9.69 s, improving his own previous world record set earlier in 2008 by 0.03 s. However, the most impressive fact about his new world record was the way in which he did it. After accelerating away from the rest of the field, he looked to his sides when 2 s and 20 m remained and started celebrating! He extended his arms, and appeared to almost dance along the track. This celebration left spectators and commentators wondering, "What would the world record have been had he not celebrated the last 20 m?" Bolt's coach, Glen Mills, later suggested that the record could have been 9.52 s, or even better.

In this paper we check this suggestion by measuring Bolt's position as a function of time and extrapolate from his dynamics before his celebration to the last 2 s of the race. Based on reasonable assumptions, we obtain an estimate of what the new world record could have been.

Earlier work on similar topics have typically revolved around the construction of a dynamical model of the 100 and 200 m dashes.[1–3] In our work we employ strictly empirical data, and fit a nonparametric model to the observations. Compared to the dynamical approaches, our analysis minimizes the number of required assumptions and makes the procedure more transparent and less prone to systematic and model-dependent errors.

II. METHOD

To make our predictions we analyze footage of the run obtained from the Norwegian Broadcasting Corporation (NRK) and from the internet (NBC and BBC). Based on these videos, we printed ≈30 screen shots at different times, from which we estimate the runners' positions as a function of time.

This task is made considerably easier by the presence of a moving camera mounted to a rail along the track (see Fig. 1). This rail is bolted to the ground at regular intervals, and thereby provides the required standard ruler. We then calibrated this standard ruler by counting the total number of bolts (called "ticks" in the following) on the rail of the moving camera along the 100 m track (see Fig. 1). We assumed the distance between the ticks to be constant.

Next, we drew lines orthogonal to the track, using whatever means was most accurate for a given screen shot. For the early and late frames, lines in the actual track (for example, starting and finishing lines) were most useful; for some intermediate frames the lower front edge of the camera mount was utilized (see Fig. 1). When reliable parallel lines were available, we used the method illustrated in Fig. 2 to estimate positions. We first found the vanishing point in the horizon where two known parallel lines appear to converge (that is, point P in the horizon in Fig. 2). Then, we drew a set of auxiliary parallel lines from this point onto the track. These lines were then propagated to earlier or later frames using fixed features in the pictures.

For a given frame we then read off the positions of Usain Bolt and Richard Thompson, the runner-up, with the ruler, and recorded their positions together with the time from the screen clock. We then assigned an uncertainty to each position measurement by estimating how many ticks we believed we were off in a given frame. For later frames, when the camera angle is almost orthogonal to the track, this uncertainty is smaller than in the beginning of the race because of the camera perspective.

Based on these uncertainties we fitted a smooth spline[4] with inverse variance weights to the data. This fit provided us with a smooth approximation to the runners' positions as a

Fig. 1. Example screen shot used to estimate the runners' position as a function of time.

function of time, from which the first and second derivatives (i.e., speeds and accelerations) may be derived.

To make the projections we consider two scenarios. We first conservatively assume that Bolt would have been able to keep up with Thompson's acceleration profile in the race after 8 s of elapsed time and project a new finishing time. Given his clearly higher acceleration around 6 s, we also consider a scenario in which he is able to maintain a $\Delta a = 0.5\ \mathrm{m/s^2}$ greater acceleration than Thompson to the finish.

The new projected world record is found by extrapolating the resulting motion profile to 100 m. We estimate the uncertainty in this number by repeating the analysis 10 000 times, each time adding a random fluctuation with specified uncertainties to each time and tick count.

III. DATA AND CALIBRATION

The data used for this analysis consist of three clips filmed by three cameras located along the finishing line at slightly different positions. Unfortunately, the NRK and BBC clips were filmed with cameras positioned fairly close to the track, and the rail of the moving camera therefore disappears outside the field of view after about 6 s. This problem does not occur for the NBC clip, which was filmed from further away. Using these data sets, we measured the position of Usain Bolt and Richard Thompson at 16 different times in units of ticks (see Table I).

There are three issues that must be addressed before the tick counts listed in Table I can be converted into proper distance measurements. First, the camera rail is not entirely visible near the starting line, because the very first part is obscured by a cameraman. The tick counts in Table I are therefore counted relative to the first visible tick. Fortunately, it is not very problematic to extrapolate into the obscured region by using the distance between the visible ticks, and knowing that the distance between the starting line for the 100 m dash and 110 m hurdles is precisely 10 m. We estimate the number of obscured ticks to be 7 ± 1.

Second, the precision of the screen clock is only a tenth of a second, and the clock appears to truncate the time, rather than to round it off. We therefore add 0.05 s to each time measurement, and assume that our time uncertainty to be uniform between −0.05 and 0.05 s.

Finally, the screen clock on the clips is not calibrated perfectly with the stadium clock (see Fig. 1 for an example frame). A little more than half of all frames appear to be synchronized, while in the rest the screen clock lags behind by 0.1 s. We assume that the stadium clock is the correct one, and recalibrate the screen clock by adding an additional 0.04 s to each time measurement. With these assumptions it is straightforward to calibrate both the clock and distance measurements, as is shown in the corresponding columns in Table I.

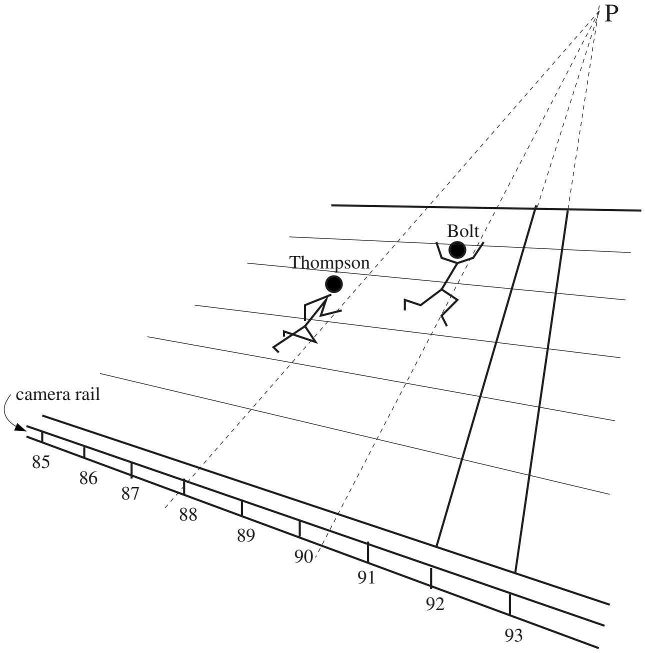

Fig. 2. Schematic illustration of the main procedure for estimating the runners' positions: Two known parallel lines, orthogonal to the track, are extended to find the horizon crossing point P for each frame. Auxilliary lines are drawn from P onto the track, corresponding to each of the runners' current positions. The positions are read off from the rail. This method was used in the second half of the race. In the first half, in which there were strong camera perspective effects, other methods were also used. See the text for details.

IV. ESTIMATION OF MOTION PROFILES

It is straightforward to make the desired projections using the calibrated distance information. First, we compute a smooth spline, $s(t)$, through each of the two runners' measured positions. A bonus of using splines is that we automatically obtain the first and second derivatives of s at each time step. To obtain a well-behaved spline we impose several constraints. We add two auxiliary data points at $t = 0.01$ s and $t = 13.0$ s. These points are included to guarantee sensible boundary conditions at each end. The first point implies that the starting velocity is zero, and the last one leads to a smooth acceleration at the finishing line. We also adopt a smooth spline stiffness parameter of $\alpha = 0.5$ to minimize unphysical fluctuations.[4] The results are fairly insensitive to the value of α.

The resulting functions are plotted in Fig. 3. Notice that Bolt and Thompson are almost neck by neck up to 4 s, corresponding to a distance of 35 m. Bolt's gold medal is essentially won between 4 and 8 s. At 8 s Bolt decelerates noticeably, and Thompson equalizes and surpasses Bolt's speed. However, Thompson is not able to maintain his speed to the finish, but slows down after about 8.5 s. Still, his acceleration is consistently higher than Bolt's after 8 s.

Table I. Compilation of distance (m) versus time (s) for Usain Bolt and Richard Thompson in the 100 m dash at Beijing 2008. The data for the second and last rows are not real observations, but ensure sensible boundary conditions for the smooth spline. The second row ensures zero starting velocity, and the last row gives a smooth acceleration at the finishing line. The data for $t=0.0$ are taken from the known starting position and therefore have zero uncertainty, and the data for $t=9.69$ and $t=9.89$ are taken from a high-resolution picture of the finish. These times are not read from the screen clock, but are adopted from official sources. Zero uncertainties are assigned to these points.

	Bolt		Thompson			
Elapsed time	Ticks	Distance	Ticks	Distance	Uncertainty	Data set
0.0	−7.0	0.0	−7.0	0.0	0.0	None
0.01	−7.0	0.0	−7.0	0.0	0.0	None
1.1	−2.0	5.0	−2.1	4.9	0.5	NRK
3.0	15.5	22.5	15.6	22.6	0.5	NRK
4.0	27.0	34.0	27.0	34.0	0.4	NRK
4.5	34.3	41.3	34.1	41.1	0.5	NRK
5.4	45.1	52.1	44.3	51.3	0.5	NBC
5.8	48.9	55.9	48.3	55.3	0.5	BBC
6.2	54.5	61.5	53.8	60.8	0.5	NBC
6.5	57.8	64.8	56.9	63.9	0.4	BBC
6.9	62.6	69.6	61.5	68.5	0.2	NBC
7.3	66.3	73.3	65.1	72.1	0.2	NBC
7.7	71.5	78.5	70.1	77.1	0.2	NBC
8.0	74.7	81.7	72.9	79.9	0.2	NBC
8.3	78.6	85.6	76.8	83.8	0.2	NBC
8.6	82.2	89.2	80.5	87.5	0.2	NBC
8.8	84.3	91.3	82.4	89.4	0.2	NBC
9.4	91.6	98.6	89.4	96.4	0.2	NBC
9.69	93.0	100.0	⋯	⋯	0.0	None
9.89	⋯	⋯	93.0	100.0	0.0	None
13	105	112	105	112	5.0	None

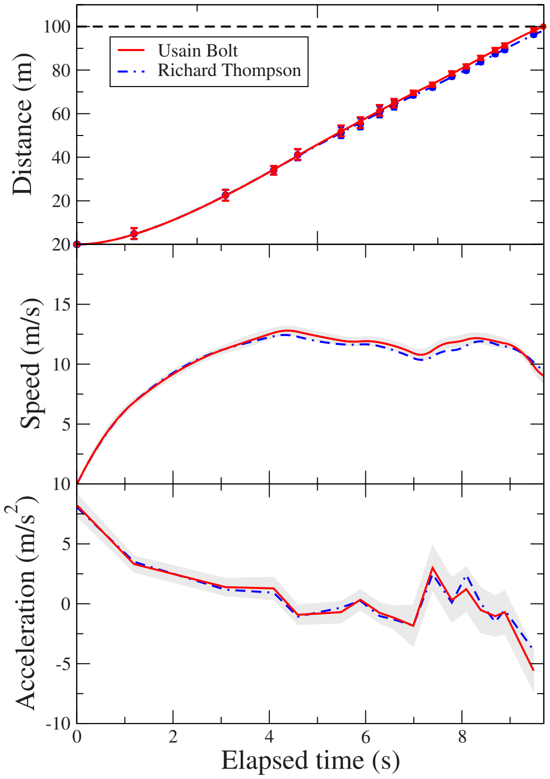

Fig. 3. Estimated position (top), speed (middle), and acceleration (bottom) for Bolt (solid curves) and Thompson (dashed-dotted curves) as a function of time. Actual distance measurements are indicated in the top panel with 5σ error bars. Gray bands indicate the 68% confidence regions for the spline estimators as estimated by Monte Carlo simulations.

It is important to note that there are significant uncertainties related to these measurements, as indicated by the 68% confidence regions (gray bands) in Fig. 3. The acceleration profile is particularly noisy (that is, it exhibits unphysical fluctuations), because it is estimated by numerically differentiating the observed distance versus time data twice.

Another point is the fact that the velocity and acceleration profiles for Bolt and Thompson correspond very closely with each other. The reason is that the two main uncertainties in determining the position and time for each runner in a given frame are common. We have to define a proper reference line normal to the track in each frame, and the screen clock has a temporal resolution of 0.1 s. These two effects result in large and common uncertainties. These errors are modeled by a Monte Carlo simulation, and the resulting uncertainties are therefore properly propagated to the final results.

V. WORLD RECORD PROJECTIONS

We can now answer the original question: How fast would Bolt have run if he had not celebrated the last 2 s? To make this projection we consider two scenarios:

(1) Bolt matches Thompson's acceleration profile after 8 s.
(2) Bolt maintains a 0.5 $\mathrm{m/s}^2$ higher acceleration than Thompson after 8 s.

The justification of the first scenario is obvious, as Bolt outran Thompson between 4 and 8 s. The justification of the second scenario is more speculative, because it is difficult to quantify exactly how much stronger Bolt was. If we look at

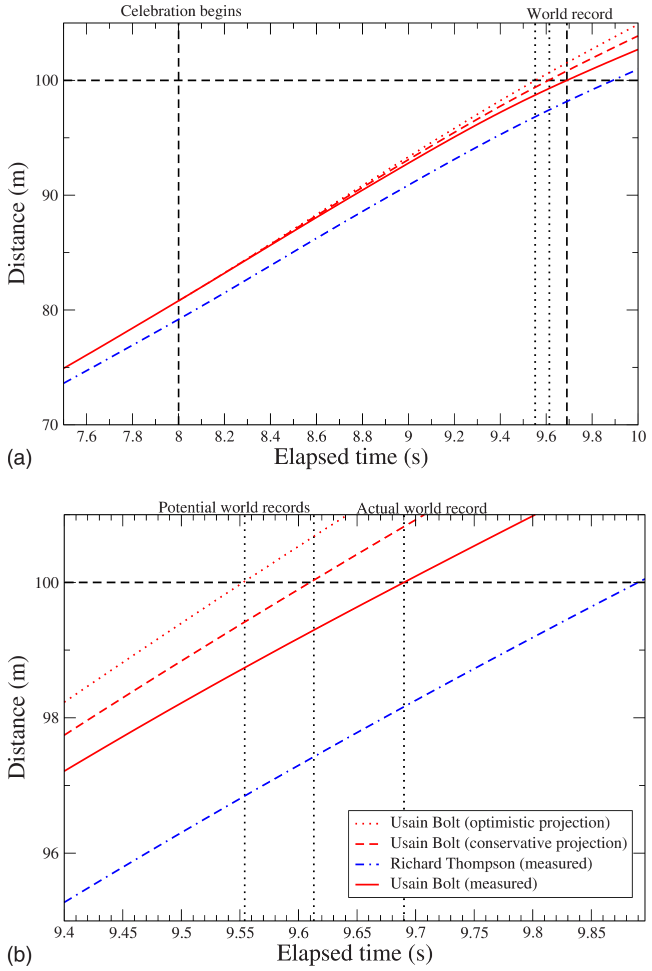

Fig. 4. (a) Comparison of real and projected distance profiles at the end of the race. The point where the profiles cross the horizontal 100 m line is the new world record for a given scenario; (b) is a zoomed version of (a). The dashed line shows the first scenario, and the dotted line shows the second scenario; the actual trajectory, $s(t)$ is shown as a solid line. For comparison, Thompson's trajectory is indicated by a dashed-dotted line.

the acceleration profiles in Fig. 3 and note that Bolt is considered a 200 m specialist, a value of $0.5\ \mathrm{m/s^2}$ seems realistic. (Negative accelerations are always observed in 100 m races; nobody is able to maintain their full speed to the finishing line.[5] Assuming for instance constant speed during the last 2 s would therefore predict an unrealistically good time.)

For each scenario we computed a trajectory for Bolt by choosing the initial conditions $s_0 = s(8\ \mathrm{s})$ and $v_0 = v(8\ \mathrm{s})$ and an acceleration profile as described. The computation of these trajectories are performed by integrating the kinematic equations with respect to time,

$$\hat{a}(t) = \begin{cases} a_{\mathrm{Thompson}}(t) & \text{(Scenario 1)} \\ a_{\mathrm{Thompson}}(t) + 0.5\ \mathrm{m/s} & \text{(Scenario 2)}, \end{cases} \tag{1}$$

$$\hat{v}(t) = v_0 + \int_{t_0}^{t} \hat{a}(t)dt, \tag{2}$$

$$\hat{s}(t) = s_0 + \int_{t_0}^{t} \hat{v}(t)dt. \tag{3}$$

In Fig. 4 we compare the projected trajectories, $\hat{s}(t)$, with the

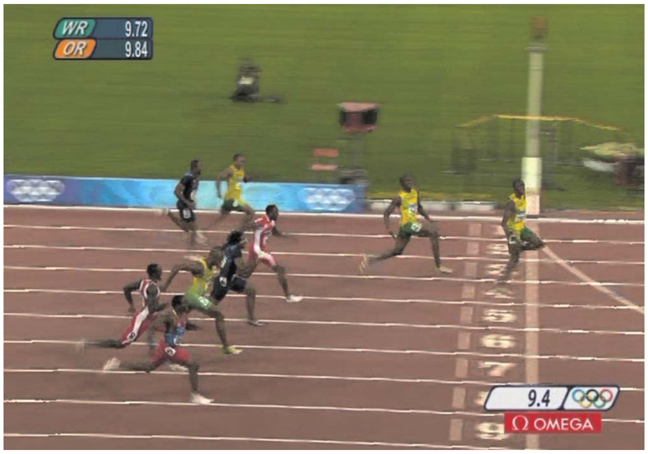

Fig. 5. Photo montage showing Bolt's position relative to his competitors for the real (left Bolt) and projected (right Bolt) world records.

actual trajectory, $s(t)$. For comparison, Thompson's trajectory is also indicated.

The projected new world record is the time at which $\hat{s}(t)$ equals 100 m. We include 95% statistical errors estimated by Monte Carlo simulations as described in Sec. II and find that the new world record would be 9.61 ± 0.04 s in the first scenario and 9.55 ± 0.04 s in the second scenario.

VI. CONCLUSIONS

Glen Mills, Usain Bolt's coach, suggested that the world record could have been 9.52 s if Bolt had not danced along the track in Beijing for the last 20 m. According to our calculations, this suggestion seems like a good, but probably an optimistic, estimate. Depending on assumptions about Bolt's acceleration at the end of the race, we find that his time would have been somewhere between 9.55 and 9.61 s, with a 95% statistical error of ±0.04 s. The uncertainties due to the assumptions about the acceleration are comparable to or larger than the statistical uncertainties. Therefore, 9.52 s is not out of reach.

In Fig. 5 we show an illustration of how such a record would compare to the actual world record of 9.69 s, relative to the rest of the field: The left version of Bolt shows his actual position at ~9.5 s, while the right version indicates his position in the new scenarios.

There are several potential systematic errors involved in these calculations. For instance, it is impossible to know for sure whether Usain Bolt might have tired at the end, which would increase the world record beyond our estimates. Judging from his facial expressions as he crossed the finishing line, this hypothesis does not strike us as very plausible.

Another issue to consider is the wind. It is generally agreed that a tail wind speed of 1 m/s improves a 100 m time by 0.05 s.[6,7] For the International Association of Athletics Federations to acknowledge a run as a record attempt, the wind speed must be less than +2 m/s. When Bolt ran in Beijing, there was no measurable wind speed, and we can therefore safely assume that the world record could have been further decreased, perhaps by as much as 0.1 s, under more favorable wind conditions.

It is interesting to note that Usain Bolt had the slowest start reaction time of the field, 0.025 s slower than Richard Thompson according to official measurements found on

IAAF's webpage. All of these issues considered together suggests that a new world record of less than 9.5 s is within reach by Usain Bolt in the near future.

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